MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 72

Total Marks : 25

Max. Time : 26 min.

Topics : Permutation & Combination, Binomial Theorem

Type of Questions		**M.M., Min.**
Comprehension (no negative marking) Q.1 to Q.3	**(3 marks, 3 min.)**	**[9, 9]**
Single choice Objective (no negative marking) Q.4	**(3 marks, 3 min.)**	**[3, 3]**
Multiple choice objective (no negative marking) Q.5	**(5 marks, 4 min.)**	**[5, 4]**
Subjective Questions (no negative marking) Q.6,7	**(4 marks, 5 min.)**	**[8, 10]**

COMPREHENSION # 1 (1 to 3)

Consider, sum of the series $\sum\sum_{0\le i<j\le n} f(i)\, f(j)$

In the given summation, i and j are not independent.

In the sum of series $\sum_{i=1}^{n}\sum_{j=1}^{n} f(i)\, f(j) = \sum_{i=1}^{n}\left(f(i)\left(\sum_{j=1}^{n} f(j)\right)\right)$ i and j are independent. In this summation, three types of terms occur, those when i < j, i > j and i = j.

Also, sum of terms when i < j is equal to the sum of the terms when i > j if f(i) and f(j) are symmetrical.

So, in that case

$$\sum_{i=1}^{n}\sum_{j=1}^{n} f(i)f(j) = \sum\sum_{0\le i<j\le n} f(i)f(j)$$

$$+ \sum\sum_{0\le i<j\le n} f(i)f(j) + \sum\sum_{i=j} f(i)f(j)$$

$$= 2\sum\sum_{0\le i<j\le n} f(i)f(j) + \sum\sum_{i=j} f(i)f(j)$$

$$\Rightarrow \quad \sum\sum_{0\le i<j\le n} f(i)f(j) = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} f(i)f(j) - \sum\sum_{i=j} f(i)f(j)}{2}$$

When f(i) and f(j) are not symmetrical, we find the sum by listing all the terms.

1. $\sum\sum_{0\le i<j\le n} {}^{n}C_i \; {}^{n}C_j$ is equal to -

(A) $\frac{2^{2n} - {}^{2n}C_n}{2}$ (B) $\frac{2^{2n} + {}^{2n}C_n}{2}$ (C) $\frac{2^{2n} - {}^{n}C_n}{2}$ (D) $\frac{2^{2n} + {}^{n}C_n}{2}$

2. $\sum_{m=0}^{n}\sum_{p=0}^{m} {}^{n}C_m \cdot {}^{m}C_p$ is equal to -

(A) $2^n - 1$ (B) 3^n (C) $3^n - 1$ (D) 2^n

3. $\sum\sum_{0\le i\le j\le n}\left({}^{n}C_{i}+{}^{n}C_{j}\right)$ is equal to -

(A) $n2^n$ (B) $(n+1)2^n$ (C) $(n-1)2^n$ (D) $(n+1)2^n-1$

4. Find the three digit numbers in which the middle one is a perfect square are formed using the digits 1 to 9 is (repeatition of digits is allowed)

(A) 243 (B) 242 (C) 244 (D) 246

5. The no. of ways in which 5 different books to be distributed among 3 persons to that each person gets at least one book, is equal to the number of ways in which

(A) 5 persons are alloted 3 different residential flats such that each person is alloted at most one flat and no two persons are alloted the some flat.

(B) No. of parallelograms formed by one set of 6 parallel lines and other set of 5 parallel lines that goes in other direction.

(C) 5 different toys are to be distributed among 3 children, so that each child gets at least one toy.

6. In how many ways can 5 colours be selected out of 8 different colours including red, blue and green

(1) if blue and green are always to be included

(2) if red is always excluded

(3) if red & blue are always included but green excluded ?

7. How many numbers between 400 and 1000 (both exclusive) can be made with the digits 2,3,4,5,6,0 if

(1) repetition of digits not allowed

(2) repetition of digits is allowed

Answers Key

1. (A)

2. (B)

3. (A)

4. (A)

5. (B)(C)

6. (1) 20 (2) 21 (3) 10

7. (1) 60 (2) 107